Question of November

Solutions

5 December, 2014

Solution of the Question 1

Let K be the set of dictinct numbers of intersections.

$$K = \{a_1, a_2, ..., a_k\}$$

Without loss of generality, we can assume $a_1 < a_2 < ... < a_k$. A red line may not be intersect a blue line and may intersect all blue lines. Thus, we can say $0 \leq a_1$ and $a_k \leq l$. Since there are k distinct numbers of intersection, we can say $k \leq l + 1$. Similarly, if we make the same assumption for blue lines, we can easily deduct $l \leq k + 1$. If we solve the equations

$$k \leq l + 1$$

$$l \leq k + 1$$

together, we have $|k - l| \leq 1$.

Solution of the Question 2

Let g_n, r_n, b_n be the numbers of the green, red and blue lizards after two lizards with different colours randomly came together n times, respectively. After $n + 1$th move, there are 3 possibility :

$$\{g_{n+1}, r_{n+1}, b_{n+1)}\} = \{g_n + 2, r_n - 1, b_n - 1\}$$

$$\{g_{n+1}, r_{n+1}, b_{n+1}\} = \{g_n - 1, r_n + 2, b_n - 1\}$$

$$\{g_{n+1}, r_{n+1}, b_{n+1}\} = \{g_n - 1, r_n - 1, b_n + 2\}.$$

In all cases, the difference between lizards with different colour doesn't change in (mod 3). Since $\{g_0, r_0, b_0\} = \{13, 15, 17\}$, it is not possible to make all lizards green, red or blue.